

Contents

<i>Preface</i>	<i>page xi</i>
0 Introduction	1
0.1 Design Problems	1
0.2 Foldability Questions	3
Part I. Linkages	
1 Problem Classification and Examples	9
1.1 Classification	10
1.2 Applications	11
2 Upper and Lower Bounds	17
2.1 General Algorithms and Upper Bounds	17
2.2 Lower Bounds	22
3 Planar Linkage Mechanisms	29
3.1 Straight-line Linkages	29
3.2 Kempe's Universality Theorem	31
3.3 Hart's Inversor	40
4 Rigid Frameworks	43
4.1 Brief History	43
4.2 Rigidity	43
4.3 Generic Rigidity	44
4.4 Infinitesimal Rigidity	49
4.5 Tensegrities	53
4.6 Polyhedral Liftings	57
5 Reconfiguration of Chains	59
5.1 Reconfiguration Permitting Intersection	59
5.2 Reconfiguration in Confined Regions	67
5.3 Reconfiguration without Self-Crossing	70
6 Locked Chains	86
6.1 Introduction	86
6.2 History	87

viii Contents

6.3 Locked Chains in 3D 88
6.4 No Locked Chains in 4D 92
6.5 Locked Trees in 2D 94
6.6 No Locked Chains in 2D 96
6.7 Algorithms for Unlocking 2D Chains 105
6.8 Infinitesimally Locked Linkages in 2D 113
6.9 3D Polygons with a Simple Projection 119

7 Interlocked Chains 123
7.1 2-chains 125
7.2 3-chains 126
7.3 4-chains 127

8 Joint-Constrained Motion 131
8.1 Fixed-Angle Linkages 131
8.2 Convex Chains 143

9 Protein Folding 148
9.1 Producible Polygonal Protein Chains 148
9.2 Probabilistic Roadmaps 154
9.3 HP Model 158

Part II. Paper

10 Introduction 167
10.1 History of Origami 167
10.2 History of Origami Mathematics 168
10.3 Terminology 169
10.4 Overview 170

11 Foundations 172
11.1 Definitions: Getting Started 172
11.2 Definitions: Folded States of 1D Paper 175
11.3 Definitions: Folding Motions of 1D Paper 182
11.4 Definitions: Folded States of 2D Paper 183
11.5 Definitions: Folding Motions of 2D Paper 187
11.6 Folding Motions Exist 189

12 Simple Crease Patterns 193
12.1 One-Dimensional Flat Foldings 193
12.2 Single-Vertex Crease Patterns 198
12.3 Continuous Single-Vertex Foldability 212

13 General Crease Patterns 214
13.1 Local Flat Foldability is Easy 214
13.2 Global Flat Foldability is Hard 217

14 Map Folding 224
14.1 Simple Folds 225
14.2 Rectangular Maps: Reduction to 1D 227
14.3 Hardness of Folding Orthogonal Polygons 228
14.4 Open Problems 230

Contents

15	Silhouettes and Gift Wrapping	232
15.1	Strip Folding	233
15.2	Hamiltonian Triangulation	233
15.3	Seam Placement	236
15.4	Efficient Foldings	237
16	The Tree Method	240
16.1	Origami Bases	240
16.2	Uniaxial Bases	242
16.3	Everything is Possible	243
16.4	Active Paths	244
16.5	Scale Optimization	246
16.6	Convex Decomposition	247
16.7	Overview of Folding	249
16.8	Universal Molecule	250
17	One Complete Straight Cut	254
17.1	Straight-Skeleton Method	256
17.2	Disk-Packing Method	263
18	Flattening Polyhedra	279
18.1	Connection to Part III: Models of Folding	279
18.2	Connection to Fold-and-Cut Problem	280
18.3	Solution via Disk Packing	281
18.4	Partial Solution via Straight Skeleton	281
19	Geometric Constructibility	285
19.1	Trisection	285
19.2	Huzita's Axioms and Hatori's Addition	285
19.3	Constructible Numbers	288
19.4	Folding Regular Polygons	289
19.5	Generalizing the Axioms to Solve All Polynomials?	290
20	Rigid Origami and Curved Creases	292
20.1	Folding Paper Bags	292
20.2	Curved Surface Approximation	293
20.3	David Huffman's Curved-Folds Origami	296
Part III. Polyhedra		
21	Introduction and Overview	299
21.1	Overview	299
21.2	Curvature	301
21.3	Gauss-Bonnet Theorem	304
22	Edge Unfolding of Polyhedra	306
22.1	Introduction	306
22.2	Evidence for Edge Unfoldings	312
22.3	Evidence Against Edge Unfoldings	313
22.4	Unfoldable Polyhedra	318
22.5	Special Classes of Edge-Unfoldable Polyhedra	321
22.6	Vertex-Unfoldings	333

x **Contents**

23	Reconstruction of Polyhedra	339
23.1	Cauchy's Rigidity Theorem	341
23.2	Flexible Polyhedra	345
23.3	Alexandrov's Theorem	348
23.4	Sabitov's Algorithm	354
24	Shortest Paths and Geodesics	358
24.1	Introduction	358
24.2	Shortest Paths Algorithms	362
24.3	Star Unfolding	366
24.4	Geodesics: Lyusternik–Schnirelmann	372
24.5	Curve Development	375
25	Folding Polygons to Polyhedra	381
25.1	Folding Polygons: Preliminaries	381
25.2	Edge-to-Edge Gluings	386
25.3	Gluing Trees	392
25.4	Exponential Number of Gluing Trees	396
25.5	General Gluing Algorithm	399
25.6	The Foldings of the Latin Cross	402
25.7	The Foldings of a Square to Convex Polyhedra	411
25.8	Consequences and Conjectures	418
25.9	Enumerations of Foldings	426
25.10	Enumerations of Cuttings	429
25.11	Orthogonal Polyhedra	431
26	Higher Dimensions	437
26.1	Part I	437
26.2	Part II	437
26.3	Part III	438
	<i>Bibliography</i>	443
	<i>Index</i>	461